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# MHD Boundary Layer Flow of a Nanofluid over an Exponentially Permeable Stretching Sheet with chemical reaction and activation energy

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## Abstract

An analysis has been carried out to investigate MHD boundary layer flow of an electrically conducting nanofluid due to an exponentially permeable stretching sheet with chemical reaction and activation energy by employing suitable numerical technique. The equations of mass, momentum, energy and nanoparticle volume fraction governing the flow of the fluid are partial differential equations which are reduced to ordinary differential equations by means of similarity transformation. These equations are solved numerically using Method of line. The non-dimensional parameters such as Brownian motion parameter Nb, Thermophoresis parameter Nt, non-dimensional energy E, Prandtl number Pr, temperature difference parameter δ, dimensionless reaction rate σ and fitted rate constant *n are* analyzed for nanoparticle volume fraction profile and the numerical values presented graphically .

***Keywords:*** *MHD; Nanofluid, Stretching permeable sheet, chemical reaction, activation energy*

## INTRODUCTION

The boundary layer flow over a stretching sheet is significant in applications**.** The analysis of boundary layer flow of viscous fluid and heat transfer due to stretching sheet has important application in industry and engineering process and polymer industry, such as in polymer extrusion drawing of copper wires, artificial fibres, paper production, wire drawing, hot rolling, glass fibres, metal exclusion and spinning of metal etc. The study of Sakiadas [1] was the first about two dimensional boundary layer flow due to stretching wire in a fluid at rest. Crane[2] analyzed the boundary layer flow due to linearly stretching sheet. An investigation of TSOU et.al [3] proved that the mathematically described boundary layer problem on a continuous moving surface is physically reasonable. Elbashbeshy [4] were studied the flow and heat transfer over an exponentially stretching surface. Magyari and Keller [5] investigated the boundary layer flow and heat transfer due to an exponentially stretching sheet. As the studies of stretching sheet garnered considerable attention, these findings prove to be crucial to these researches.

Partha et.al [6] investigated a similarity solution for mixed convection flow past an exponentially stretching surface. Ishak [7] presented the magneto hydrodynamics (MHD) boundary layer flow over an exponentially shrinking sheet in presence of thermal radiation. Bhattacharyya [8] analyzed the boundary layer flow and heat transfer caused due to exponentially shrinking sheet. Kishan and Kavitha[9] discussed MHD Non-Newtonian power Law Fluid flow and Heat Transfer past a Non-Linear stretching surface . Bhattacharyya ad pop [10] studied the effects of external magnetic field on the flow on the flow over an exponentially shrinking sheet.

In certain porous media applications, working fluid heat generation (source) or absorption(sink), chemical reaction and activation energy effects are important. Many authors like Gupta and Sridhar [11], Abel and Veen [12] and Sharma [13] had given their considerable attention to analyze these effects. Choi[14] was the first person who introduced the term nanofluid which contains nano mater sized particle fibres suspended in the base fluid. The nanofluid is an advance type of nanofluids was observed by Masuda et al.[15]. Buongiorno[16] explained the reasons behind the enhancement in heat transfer for nanofluid and he found that Brownian diffusion and thermophoresis are the main causes. Later, NieldndKuznetsov[17], Kuznetsov and Nield[18] studied the natural convective boundary layer flow of a nanofluid employing Buongiorno model.

Rosmilla[19] et al. theoretically investigated the problem of steady boundary layer flow of nanofluid past of nanofluid past a porous stretching surface with variable stream conditions and chemical reaction. Rosca[20] et al. studied steady forced convection stagnation point flow and mass transfer past a permeable stretching/shrinking sheet placed in a copper (cu) water based nanofluid. Khan and Pop[21] firstly explored the boundary layer flow of nanofluid past a linearly stretching sheet and introduced the model of Nield and Kuznetsov[22]. Makinde and Aziz[23] explained the boundary layer flow induced in a nanofluid due to a linearly stretching sheet with convective boundary condition. MHD boundary layer flow of a nanofluid past a vertical stretching permeable surface with suction/injection was described by Kandasamyet *et al.* [24]. Rana and Bhargava[25] considered the steady, laminar boundary layer flow due to the nonlinear stretching of a flat surface in a nanofluid. Hady *et al.*[26] extended the boundary layer flow and heat transfer characteristics of a viscous nanofluid over a nonlinearly stretching sheet in the presence of thermal radiation and variable wall temperature. Hunegnaw and NaikotiKishan[27] illusrated the MHD boundary layer flow and heat transfer of a nanofluid past a non-linearly permeable stretching/shrinking sheet with thermal radiation and suction effects in the presence of chemical reaction. Kishan *et al* [28] discussed MHD Boundary Layer Flow and Heat Transfer of a Nanofluid Over a shrinking Sheet with Mass Suction and Chemical Reaction.

Krishnedu Bhattacharyya and Layek[29] explored MHD boundary flow of nanofluid due to an exponentially permeable stretching sheet. The study of mass transfer is the important phenomenon in many processes such as thermal insulation, food processing, absorption and condensation in a mixture. In this present study, we investigated the influence of thermal radiation and internal heat generation on magneto hydrodynamics boundary layer flow of nanofluid due to an exponentially

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stretching permeable sheet with the effects of chemical reaction and activation energy on nanoparticle volume fraction. The governing partial differential equations are reduced to ordinary differential equations and then solved numerically using Method of line by Mathematica. The effects of non-dimensional parameters are presented graphically.

## MATHEMATICAL FORMULATION

We consider the two dimensional steady boundary layer flow of a nanofluid over an exponentially stretching sheet in presence of a transverse magnetic field. The velocity components u and v are taken in X and Y directions respectively. The system of governing equations of motion and the energy in the presence of binary chemical reaction with Arrhenius activation energy may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho_f}u \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{(\rho c)_p}{(\rho c)_f}\left[D_B\frac{\partial N}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]-\frac{1}{(\rho c)_p}\frac{\partial q_r}{\partial y} \qquad (3)$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=D_B\frac{\partial^2 N}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-K_r^2(N-N_\infty)\left(\frac{T}{T_\infty}\right)^n e^{\left(-\frac{E_a}{\kappa T}\right)} \qquad (4)$$

Where $\upsilon$ is the kinematic Viscosity, $\rho_f$, is the density of the base fluid, T is the temperature, $T_\infty$ is constant temperature of the fluid in the inviscid free stream, α is the thermal conductivity, $(\rho c)_p$ is the effective heat capacity of nanoparticles, $(\rho c)_f$ is heat capacity of the base fluid, N is nanoparticle volume fraction, $D_B$ is the Brownian diffusion coefficient and $D_T$ is the thermo phoretic diffusion coefficient. The term $K_r^2(\phi-\phi_\infty)\left(\frac{T}{T_\infty}\right)^n e^{\left(-\frac{E_a}{\kappa T}\right)}$ in equation (4) represents the modified Arrhenius equation in which $K_r^2$ is the reaction rate, $E_a$ the activation energy, $\kappa$ = 8.61x10$^{-5}$eV/K the Boltzmann constant and n fitted rate constant which generally lies in the range -1 < n < 1.

Here, the variable magnetic field B(x) is of the form $B(x) = B_0 \exp\left(\frac{x}{2L}\right)$, where $B_0$ is a constant. Using Rosseland approximation for radiation:

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y'} \quad (5)$$

Where k $^*$ is the mean absorption coefficient and σis the Stefan Boltzmann constant. $T^4$ is expressed as a linear function of temperature by using Taylor series expansion about $T_\infty$ is: $T^4 = 4T_\infty^3 T - 3T_\infty^4$

The boundary conditions of the above system are given by

$$u = U_w(x), v = v_w \text{ at } y = 0, u \to 0 \text{ as } y \to \infty,$$

$$T = T_w = T_\infty + T_0 \exp\left(\frac{x}{2L}\right) \; at \; y = 0, T \to T_\infty \; as \; y \to \infty, \quad (6)$$

$$N = N_w = N_\infty + N_0 \exp\left(\frac{x}{2L}\right) at \; y = 0, N \to N_\infty \; as \; y \to \infty,$$

Where $T_w$ is the variable temperature at the sheet with $T_0$ being a constant which measures the rate of temperature increase along the sheet, $N_w$ is the variable wall nanoparticle volume fraction with $N_0$ being a constant and $N_\infty$ is constant nanoparticle volume fraction in free stream. The stretching velocity $U_w$ is given by

$$U_w(x) = c\exp\left(\frac{x}{L}\right), \quad (7)$$

here $c > 0$ is stretching constant. Here $v_w$ is the variable wall mass transfer velocity and is given by

$$v_w(x) = v_0 \exp\left(\frac{x}{L}\right) \quad (8)$$

where $v_0$ is the constant with $v_0 < 0$ for mass suction and $v_0 > 0$ for mass injection.

To solve the governing equations (2) to (4) under the boundary conditions (6),we introduce the similarity transformations:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \psi = \sqrt{2vLc}\, f(\eta)\exp\left(\frac{x}{2L}\right), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (9)$$

$$\varphi(\eta) = \frac{N - N_\infty}{N_w - N_\infty}, \eta = y\sqrt{\frac{c}{2vL}}\exp\left(\frac{x}{2L}\right)$$

The equations (2) to (4) can be transformed in to the following equations:

$$f''' + ff'' - 2f'^2 - Mf' = 0 \quad (10)$$

$$\left(1+\frac{4}{3}R\right)\theta''+\Pr\left(f\theta'-f'\theta+Nb\theta'\varphi'+Nt\theta'^{\frac{3}{2}}\right)=0$$

(11)

$$\varphi''+Le\left(f\varphi'-f'\varphi\right)+\frac{Nt}{Nb}\theta''-Le\sigma(1+\delta\theta)^{n}\varphi e^{\left(-\frac{E}{1+\delta\theta}\right)}=0$$

(12)

Where $M=2\sigma B_0^2L/c_\rho$ is the magnetic parameter, $Pr=\upsilon/\alpha$ is the Prandtl number and $Le=\upsilon/D_B$ is the Lewis number, $R=\frac{4\sigma T_\infty^{\ 3}}{kk^*}$ is the Radiation parameter. The dimensionless parameter Nb (Brownian motion parameter) and Nt (thermophoresis parameter) are defined as

$$Nb=D_B\frac{(\rho c)_p}{(\rho c)_f}\frac{(N_w-N_\infty)}{\upsilon},\ Nt=\frac{D_T}{T_\infty}\frac{(\rho c)_p}{(\rho c)_f}\frac{(T_w-T_\infty)}{\upsilon},\ R=\frac{4\sigma T_\infty^{\ 3}}{kk^*}$$

$$\sigma=\frac{K_r^2}{c}\quad \delta=\frac{T_w-T\infty}{T\infty}\quad E=\frac{E_a}{\kappa T}$$

The boundary conditions (6) reduce to the following forms:

$$f(\eta)=S,\ f'(\eta)=1\ at\ \eta=0$$
$$f'(\eta)\to 0\ as\ \eta\to\infty,$$
$$\theta(\eta)=1\ at\ \eta=0,\ \theta(\eta)\to 0\ as\ \eta\to\infty$$
$$\varphi(\eta)=1\ at\ \eta=0, \varphi(\eta)\to 0\ as\ \eta\to\infty$$

(13)

Where $S=-v_0/\sqrt{\upsilon c/2L}$ is the wall mass transfer parameter, $S>0(v_0<0)$ corresponds to mass suction and $S<0(v_0>0)$ corresponds to mass injection.

**RESULTS AND DISCUSSION**

The present section, in order to get a clear insight of the physical problem, profile of nanoparticle volume friction have been established by assigning numerical values to the governing parameters like Brownian motion parameter (Nb), Thermophoresis parameter (Nt), Prandtl number (Pr), non-dimensional energy (E),temperature difference parameter (δ), dimensionless reaction rate (σ) and fitted rate constant (*n)*. The Numerical computations are presented graphically from figures.1-7.

Fig.1 displays the effects of Prandtl number (Pr) on the nanoparticle volume fraction. The nanoparticle volume fraction increases with an increasing value of Prandtl number (Pr).

Fig.2 depicts the effects of Brownian motion parameter (Nb) on the nanoparticle volume fraction. The nanoparticle volume fraction profile decreases with an increasing value of Brownian motion parameter (Nb).

Fig.3 demonstrates the effects of Thermophoresis parameter (Nt) on the nanoparticle volume fraction. It reveals that the nanoparticle volume fraction increases with an increasing value of Thermophoresis parameter (Nt).

Fig.4 illustrates the effects of non-dimensional energy (E) on the nanoparticle volume fraction. The nanoparticle volume fraction increases with an increasing value of non-dimensional energy (E) which enhances nanoparticle volume fraction boundary layer thinness.

Fig.5 depicts the effects of temperature difference parameter (δ) on temperature and the nanoparticle volume fraction. The nanoparticle volume fraction profiles decreases with an increasing values of temperature difference parameter (δ) which decreases nanoparticle volume fraction boundary layer thickness.

Fig.6 displays the effects of dimensionless reaction rate (σ) on temperature and the nanoparticle volume fraction. The nanoparticle volume fraction profile decreases with an increasing value of dimensionless reaction rate (σ) within boundary layer region.

Fig.7 represents the effects of and fitted rate constant (*n*) on the nanoparticle volume fraction. The nanoparticle volume fraction profile decreases with an increasing value of fitted rate constant (*n)* which leads to considerable thinning with in the boundary layer.

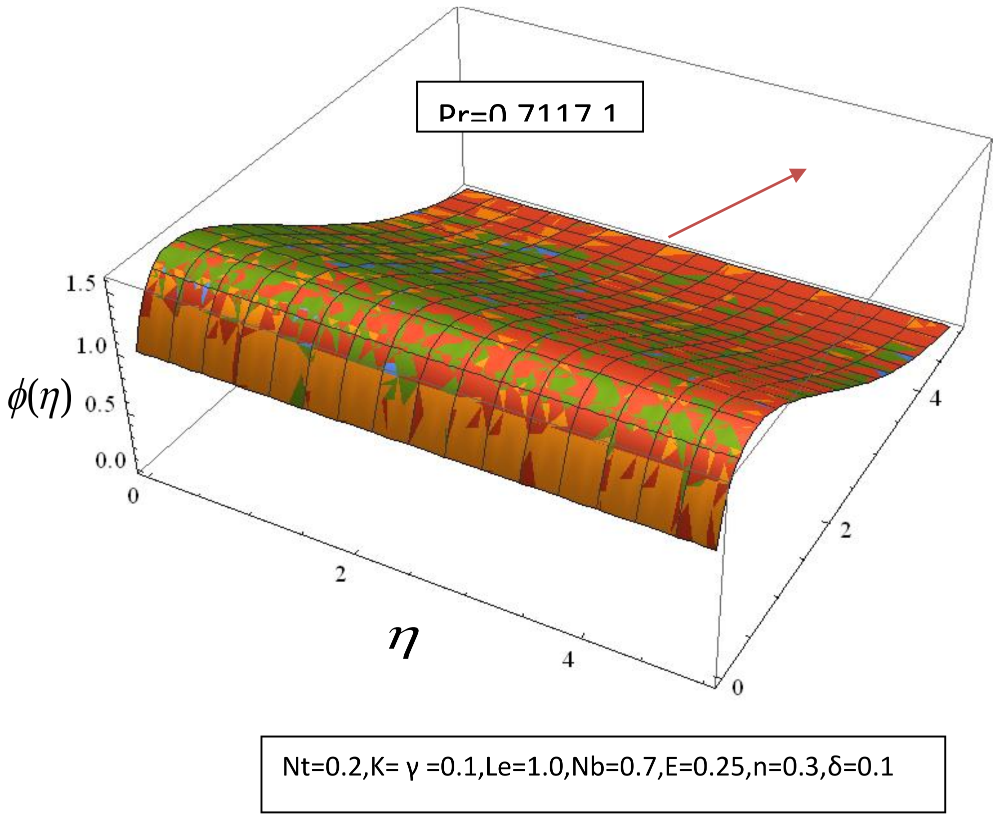


**Fig 1: Effects of Prandtl number Pr on nanoparticle volume fraction**

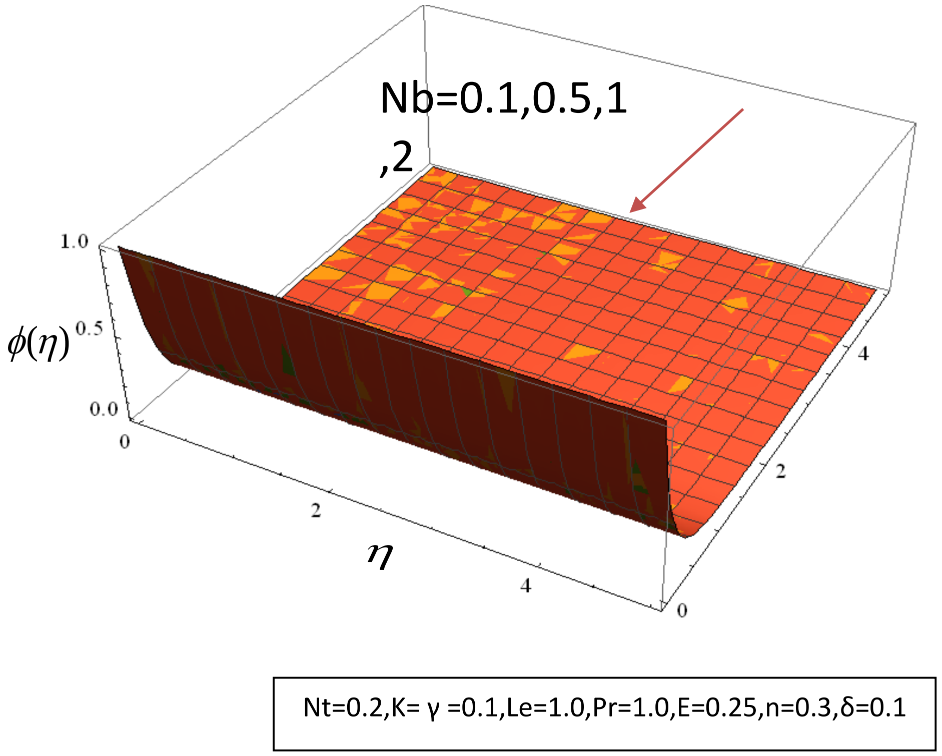


**Fig 2: Effects of Brownian motion parameter Nb on nanoparticle volume fraction**

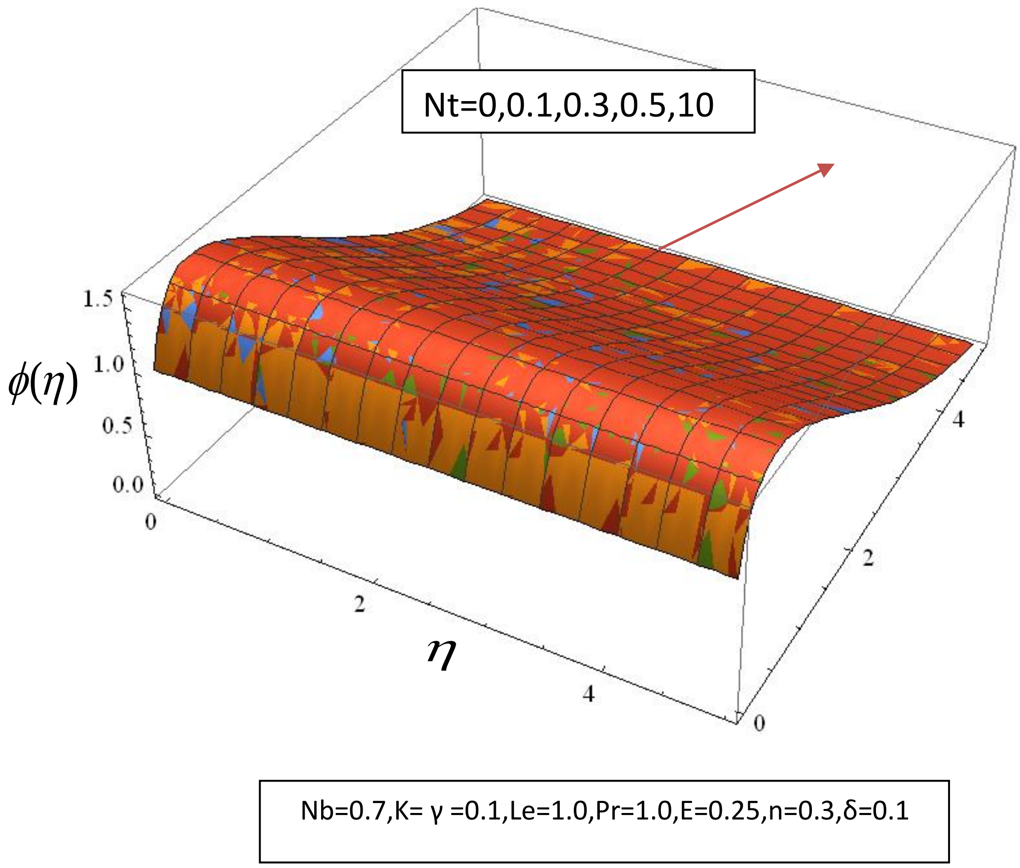


**Fig 3: Effects of Thermophoresis parameter Nt on nanoparticle volume fraction**

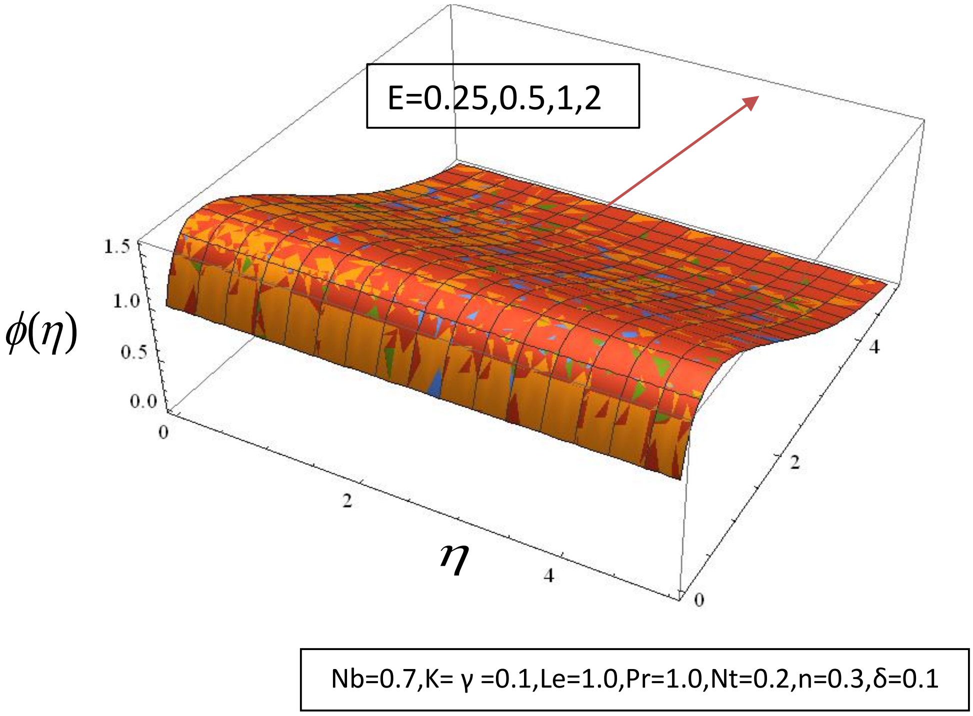


**Fig 4: Effects of non-dimensional energy E on nanoparticle volume fraction**

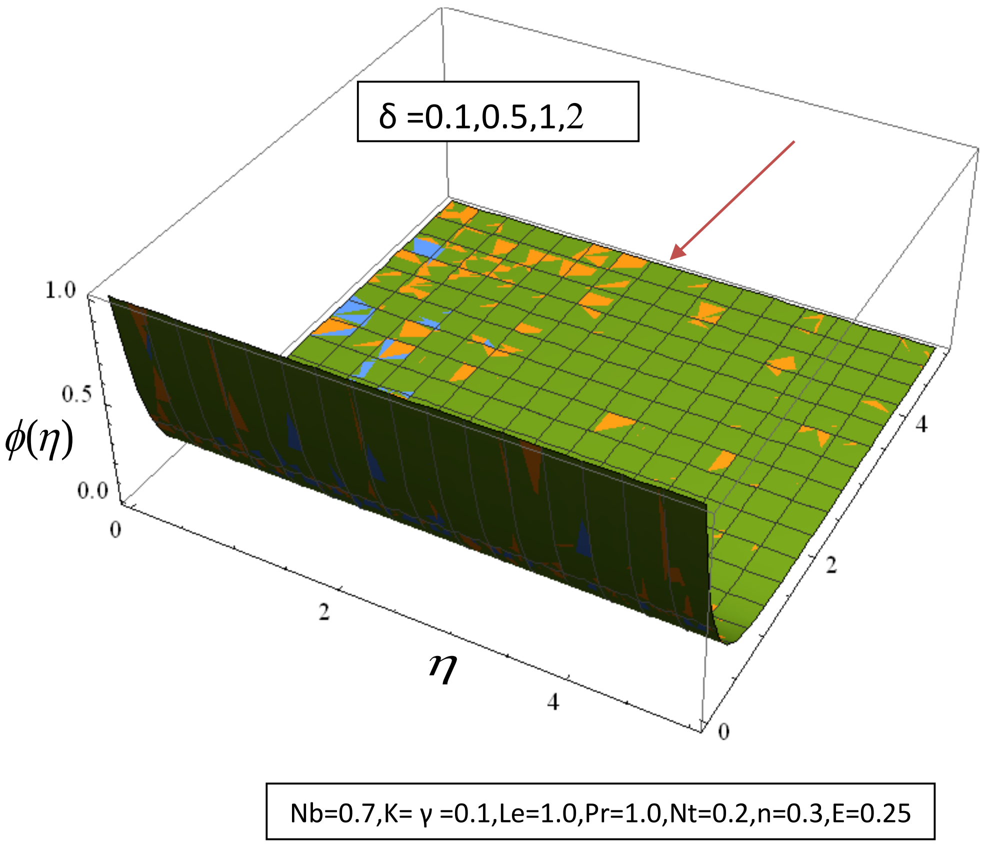


**Fig 5: Effects of temperature difference parameter δ on nanoparticle volume fraction**

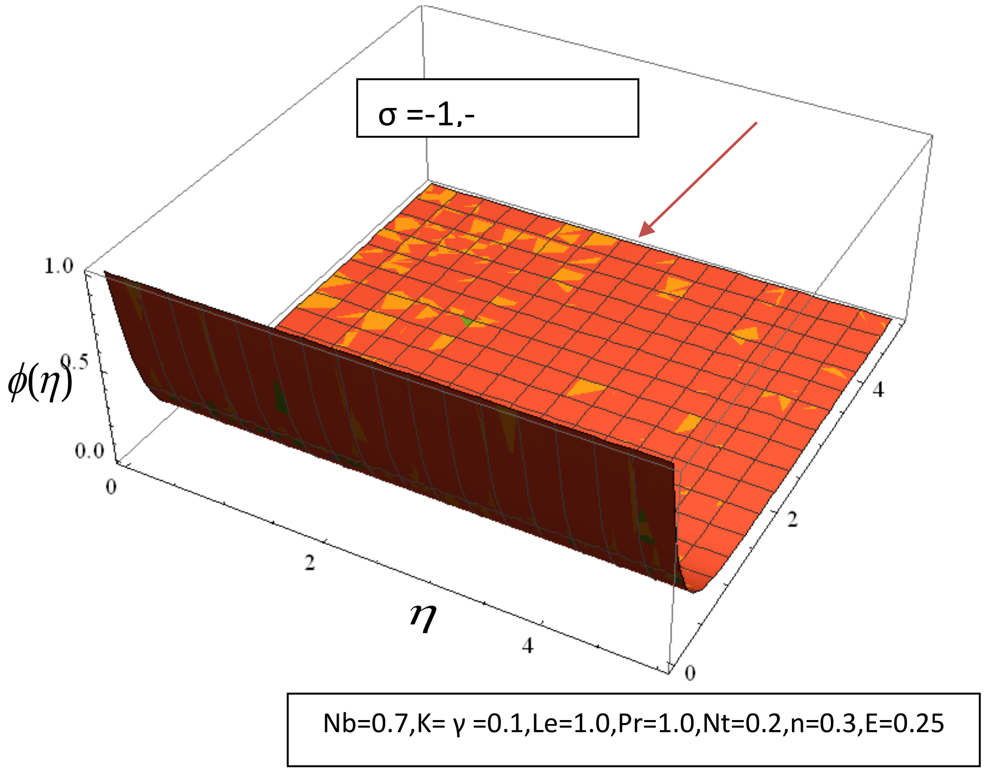


**Fig 6: Effects of dimensionless reaction rate σ on nanoparticle volume fraction**

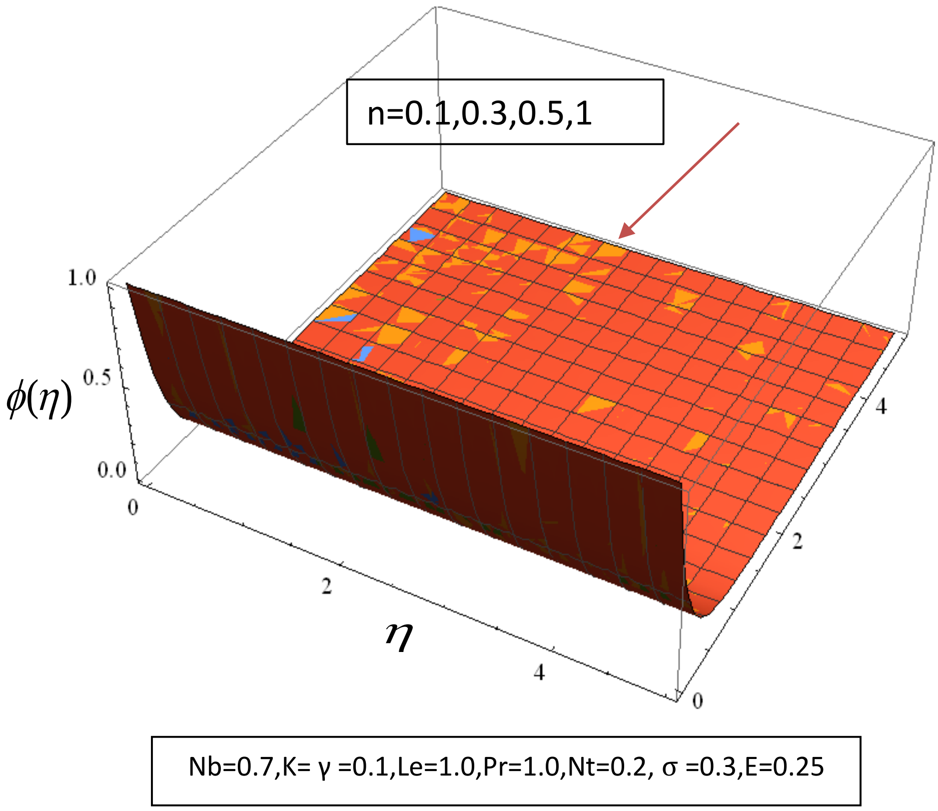


**Fig 7: Effects of fitted rate constant *n* on nanoparticle volume fraction**

## CONCLUSIONS

In this investigation, MHD boundary layer flow of an electrically conducting nanofluid with chemical reaction and activation energy due to an exponentially permeable stretching sheet has been considered. The effect of governing non-dimensional parameters such as Brownian motion parameter Nb, Thermophoresis parameter Nt, non-dimensional energy E, Prandtl number Pr, temperature difference parameter δ, dimensionless reaction rate σ and fitted rate constant *n on* nanoparticle volume fraction presented graphically.

- The nanoparticle volume fraction increases with an increasing value of Prandtl number (Pr).
- The nanoparticle volume fraction profile decreases with an increasing value of Brownian motion parameter (Nb).
- It reveals that the nanoparticle volume fraction increases with an increasing value of Thermophoresis parameter (Nt).
- The nanoparticle volume fraction increases with an increasing value of non-dimensional energy (E) which enhances nanoparticle volume fraction boundary layer thinness.
- The nanoparticle volume fraction profiles decreases with an increasing values of temperature difference parameter (δ) which decreases nanoparticle volume fraction boundary layer thickness.
- The nanoparticle volume fraction profile decreases with an increasing value of dimensionless reaction rate (σ) within boundary layer region.
- The nanoparticle volume fraction profile decreases with an increasing value of fitted rate constant (*n)* which leads to considerable thinning with in the boundary layer.

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